Deep Inference and the Calculus of Structures

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http://alessio.guglielmi.name/res/cos .

1 Brief Overview

The *calculus of structures* is a new proof theoretical formalism, introduced by myself in 1999 and initially developed by members of my group in Dresden since 2000. It exploits a new *symmetry* made possible by *deep inference*. We can present deductive systems in the calculus of structures and analyse their properties, as we do in the sequent calculus, natural deduction and proof nets. Typical properties of interest are *normalisation* and *cut elimination*. There are now many researchers around the world developing deep inference and investigating its consequences on proof theory. This document gives an overview of this research effort.

The main purpose of our new formalism is to allow a *richer combinatorial analysis* of proofs than the other formalisms do. We adopted two main ideas: inferences are *symmetric* between premises and conclusions, and they are deeply applicable inside expressions, what we call *deep inference*. As a consequence, it is convenient to deduce over *structures*, which are expressions intermediate between formulae and sequents.

The calculus of structures generalises straightforwardly the sequent calculus, but it allows more freedom in the design of deductive systems. We can in fact design systems where all rules are *atomic*, or *local*, including contraction, promotion, all the additive rules and, most importantly, the cut rule. In particular, the cut rule gets decomposed into several rules, all *independently* admissible.

The proof theory of the calculus of structures *differs significantly* from what has been studied so far. Deep inference severely challenges the usual cut elimination methods, but this is mitigated by atomic cuts being much simpler objects than generic cuts.

We developed several new techniques to overcome the difficulties. Some of these methods allowed us to unveil totally new combinatorial properties of logical systems, like the *decomposition* theorems. In general, the new properties have a strong flavour of *modularity*. This, together with atomicity and locality, is important for applications in computer science, which we recently started to pursue.

We presented in the calculus of structures *classical logic*, *intuitionistic logic*, *modal logic*, *linear logic* and several *non-commutative linear logics*, in such a way that atomicity, locality and the various instances of modularity can been expressed. The deductive systems in the calculus of structures are *simple* and *elegant*, and they pose no special challenge to those interested in using them. Moreover, the calculus of structures is inspiring a *new generation of proof nets* with better proof theoretic qualities, and we are addressing important issues like the problem of *identity of proofs*.

This whole theory was born when trying to capture a mixed commutative/non-commutative logic, which I call system V. Only the multiplicative fragment of V has been developed so far—again with all the good properties mentioned above. Despite its simplicity (only three inference rules), this system *cannot be presented* in the sequent calculus. This is rather surprising and allows us to claim our ability *to extend* the range of application of the sequent calculus, the most versatile formalism so far.

Most of this document is a guide to the literature produced by our group in Dresden, in Sect. 2. In Sect. 3 I give some brief indications of the activity around deep inference, which is taking place outside of Dresden, for what I am aware of. Paola Bruscoli and I recently moved to the University of Bath, and we intend to build a strong group there, too. This document will gradually become less Dresden-centric in the near future, especially because I intend to be more specific in the final parts, which are for now too sketchy. In any case, the references section contains all and only the literature on deep inference that I am aware of, and some brief indications about its contents are given in the text.

2 Achievements and Recent Research Themes in Dresden

The calculus of structures has been discovered while looking for a logical system, originally chased inside the sequent calculus[1] of linear logic[2], able to make commutative and non-commutative multiplicative logical relations coexist. Our constraints were:

1 only one non-commutative self-dual connective should exist;

2 the cut rule should be admissible.

Constraint 2 is obviously due to the desire of making some meaningful and rich proof theory, while constraint 1 stems from the desire to capture sequential composition (like in CCS) in proof construction. This problem was widely recognized as very hard, and in fact baffled me for several years, until it was solved in (a manuscript that later became) [17].

The main idea of the calculus of structures, made possible by deep inference, is as follows. In the sequent calculus, where only shallow inference is available, derivations are trees; in the calculus of structures, they are *superpositions* of trees, which can also be flipped upside-down. This may sound like a complex idea, but it is actually very simple if we merge the coding of trees, sequents and formulae into a single kind of expression, which we call *structure*, and which is essentially a sequent, as in the tradition. Systems in the calculus of structures have all the symmetries of the sequent calculus, plus a *top-down symmetry*, which is new and is responsible for *modularity* in the cut elimination argument, which is a methodological achievement of ours. It soon became clear that the calculus of structures has a very broad use, namely it is applicable to all logics that admit an involutive negation (like classical logic and linear logic). The calculus of structures was later adapted to intuitionistic logics, where negation is not involutive.

The cut rule, which is a general form of the venerable modus ponens, is the totem of proof theory. Modus ponens is shallow, it joins two *separated* derivations with matching endpoints. Totems are usually associated to taboos, so there is some work to do in order to convince proof

[1] Gerhard Gentzen. Investigations into logical deduction. In M. E. Szabo, editor, *The Collected Papers of Gerhard Gentzen*, pages 68-131. North-Holland, Amsterdam, 1969.

[2] Jean-Yves Girard. Linear logic. *Theoretical Computer Science*, 50:1-102, 1987.

theorists to use deep forms of cut. There has been an attempt by Schütte in the 60's[3], but his form of deepness didn't go as far as dropping the requirement for the two derivations to be separated, and what he got was a proof theory which only marginally improved on the traditional one. In the last few years in Dresden we showed how the shallow cut is just a special instance of the deep cut, and that there are several advantages in moving to deep cut, if one is interested in designing deductive systems with good combinatorial properties.

Our group grew progressively:

- Lutz Straßburger (PhD student, supervised by myself) joined in July 2000; he left in September 2003, after defending his PhD thesis, for which he got the maximum grade, to join INRIA's Project Calligramme as a post-doc; after a period as researcher in the Programming Systems Lab at Universität des Saarlandes in Saarbrücken, he is now a researcher at the École Polytechnique in Paris;
- Alwen Tiu (MSc student, supervised by myself) joined in August 2000; he left in August 2001, after defending his MSc thesis, for which he got the maximum grade, to join Dale Miller's group at Penn State (and later at École Polytechnique) as a PhD student; after being a post-doc at INRIA, he is now a researcher at the Australian National University in Canberra;
- Kai Brünnler (PhD student, supervised by myself) joined in November 2000; he left in October 2003, after defending his PhD thesis, for which he got the maximum grade, to join Gerhard Jäger's group in Bern as a researcher, position he currently holds;
- Paola Bruscoli (researcher) joined in February 2001;
- Charles Stewart (researcher) joined in July 2002;
- Phiniki Stouppa (MSc student, supervised by Charles Stewart) joined in November 2002; she left in October 2004, after defending her MSc thesis, for which she got the maximum grade, to join Gerhard Jäger's group in Bern as a PhD student;
- Ozan Kahramanoğulları (PhD student, supervised by Steffen Hölldobler) joined in February 2003;
- Robert Hein (MSc student, supervised by Charles Stewart) joined in November 2003; he defended his MSc thesis in March 2005, and he got the maximum grade for it.

In the following, I present a subsection for each of the branches of research where we currently have publications. At http://alessio.guglielmi.name/res/cos one can find our main web page. There is also a mailing list devoted to the calculus of structures, structads and proof nets, which I moderate; it currently has 74 subscribers and it is reachable from http://www.prooftheory.org/frogs.

I keep a web page about the current research topics and open problems in deep inference, at http://alessio.guglielmi.name/res/cos/crt.html. This page contains information about papers currently under development, what is not included here.

While reading the rest of this document, it is important to bear in mind a terminological distinction we make: a *(formal, or deductive) system* is a set of inference rule schemes, a *calculus* is the formalism in which the formal systems are designed. For example, system LK is a deductive system for classical logic in the sequent calculus.

2.1 Classical Logic (Kai Brünnler, Alessio Guglielmi and Alwen Tiu)

This is probably the most ambitious part of our research, regarding the importance of our approach

[3] Kurt Schütte. *Beweistheorie.* Springer-Verlag, 1960.

for the foundations of proof theory. Has the calculus of structures something profound to teach us about cut elimination and the problem of identity of proofs?

This whole research project considers cut elimination in all systems for which it is known, but the main one is, of course, classical logic. One avenue for getting cut elimination is preliminary proving decomposition theorems, which are special normal forms in which the cut rule gets confined to special places inside derivations. It turns out that getting decomposition theorems for classical logic inside the calculus of structures is the most difficult case so far. It is possible to do so semantically, i.e., non-constructively, but we did not find a way to do so by normalisation.

It should be noted that in [3] Kai Brünnler showed that the sequent calculus cannot exhibit a decomposition property. Even in the absence of a decomposition theorem, we can anyway prove cut elimination by resorting to the special case of the sequent calculus [10, 6]. This is a way of getting the result, but it is certainly not satisfactory, because the final goal is to get proofs of cut elimination which completely belong to the formalism of the calculus of structures.

Thanks also to a crucial insight of Alwen Tiu, classical logic can be shown to admit an entirely local presentation in the calculus of structures. This result opens the way to a satisfactory proof of cut elimination: a new proof of cut elimination for classical logic, relying on locality, is shown in [2]. It is, to our knowledge, the most compact and elegant syntactical proof of cut elimination.

We are currently writing down another proof of cut elimination for classical logic, this time based on a splitting theorem [17] (see later about this notion). The new proof yields a finer control on the blowing up of proofs during normalisation than both the sequent calculus and the procedure in [2].

In [8, 7] we show a simple way of making classical logic finitary, without having to resort to cut elimination. It is a rather straightforward consequence of the possibility of reducing cuts to atomic form, which is impossible in the sequent calculus, outside of a standard cut elimination procedure.

A special feature of the calculus of structures, particularly in conjunction with relation web semantics [17] (see later), is the possibility of driving the search for a proof by matching pairs of atoms. A local presentation of classical logic, together with this feature, should find applications in such areas as automated reasoning (there could be relations to Bibel's connection method) and (abstract) logic programming.

Most results in this thread are contained in the thesis [4] (supervised by myself), which has been successfully discussed on September 22, 2003, and received a grade of 'summa cum laude' (the highest possible).

2.2 Modal Logic (Robert Hein, Charles Stewart, Phiniki Stouppa and Kai Brünnler)

Most of the following results are contained in Phiniki Stouppa's and Robert Hein's master theses (both supervised by Charles Stewart), respectively defended in October 2004 and in March 2005 [42, 23]

In [41, 42, 43] one finds a systematic presentation for the systems K, D, M, S4 and S5 in the calculus of structures. Thanks to deep inference, it is possible to axiomatise the modal logics in a manner directly analogous to the Hilbert axiomatisation.

This is remarkable because the usual structural proof theory of modal logics, as given in the sequent calculus, is ad-hoc; it lacks uniformity, in contrast to the pleasant situation in the semantics of modal logics, which is instead systematic. Also, we give a uniform semantic proof of cut admissibility for many of these systems. It should be noted that there is no cut-free system for S5 in the sequent calculus.

Other known formalisms with S5 presentations make use of restricted forms of deep inference. In particular, also for proving cut elimination for S5, we have established a connection to the formalism of hypersequents [43].

In [23] a method is introduced to characterise frame relational properties by means of deep inference. The achievements of this thesis are the use of a technique from modal model theory, known as unfolding (or unravelling), to characterise modal logics in a labelled sequent calculus that considers only tree-like structures. As this unfolding quickly becomes complex, Hein identifies a class of modal logics, called 3/4-Scott-Lemmon, for which unfolding remains simple. This includes important logics such as K, T, B, S4 and S5. He gives a cut-free translation from the labelled system into the calculus of structures. See also [24].

We are exploring the connections between our formalism and Belnap's display calculus, which also adopts a form of deep inference encapsulated inside a normal (shallow inference) form of sequents. The display calculus is very successful at dealing with modal logics, but the calculus of structures provides some dramatic simplifications and technical advantages.

Future directions of research are about proving cut elimination internally to the calculus of structures, via splitting theorems, and semantically, by extending the existing semantic proofs to other systems. Finding simple presentations for systems like B is also going to be a primary objective, given that no satisfying deductive system exists for them.

Recently, Kai Brünnler introduced a deep inference kind of sequent calculus (called *deep sequents*) that axiomatises modal logics previously only obtainable by the use of labels [1]. These new techniques allow to obtain pure syntactic systems with several appealing properties, which previously escaped a systematic and coherent presentation in formalisms without deep inference.

2.3 Linear Logic (Alessio Guglielmi and Lutz Straßburger)

In the sequent calculus of linear logic, the times rule suffers from a certain exponential combinatorial complexity in its proof-construction application. As a byproduct, in [17], I obtained instead that the corresponding of the times rule, in the calculus of structures, did not exhibit the same problem. This stems from the more liberal applicability of the rules in the calculus of structures, and it is natural to extend this investigation to all of linear logic.

The results in [18, 47] are the following:

1 In the calculus of structures, the promotion rule of multiplicative exponential linear logic becomes local, too, contrary to what happens in the sequent calculus.

2 Cut elimination can be proved through the use of a decomposition theorem, plus splitting.

Result 1 is particularly exemplary of the flexibility of our calculus. Result 2 is not strictly necessary to the case at hand, since cut elimination might be obtained indirectly from the sequent calculus of linear logic. It is very important, though, in view of the conservative extension of multiplicative exponential linear logic with the non-commutative operator of [17].

Cut elimination is obtained with help from a *decomposition* theorem, which is the possibility of restructuring any derivation into the composition of several independent derivations obtained in disjoint fragments of the original system (this property can hardly be expressed in the sequent calculus and does not reach the level of flexibility we get in our formalism).

In [44, 45] Lutz Straßburger presents a system comprising all of linear logic, in which *every* rule is local, including contraction. Since linear logic is a very rich system, where several subsystems correspond to computational mechanisms of varied nature, this result should have a large impact on applications, including the realm of proof-normalisation, where the non-locality of contraction is one of the major roadblocks.

Results in this thread are contained in the thesis [46] (supervised by myself), which has been successfully discussed on July 24, 2003, and received a grade of 'summa cum laude' (the highest possible).

2.4 Commutative/Non-commutative Linear Logic (Alessio Guglielmi, Ozan Kahramanoğulları, Lutz Straßburger and Alwen Tiu)

System Design

In [17, 18] we present a very simple system, called BV (and, formerly, MV), which features one self-dual, non-commutative, multiplicative connective together with two mutually dual, commutative, multiplicative ones. The system enjoys cut elimination and the argument for proving this property is modular. Cut elimination is obtained through a *splitting* theorem; this is a new, powerful, general technique introduced in [17] for deep inference deductive systems, which found applications in all deductive systems so far conceived within the calculus of structures.

We considered the problem of conservatively extending BV to exponential modalities. For BV, Alwen Tiu showed that there is no corresponding sequent system to rely upon (see [58, 55] and below). When no corresponding system exists in the sequent calculus, like in this case, the cut elimination argument becomes very delicate. System BV is NP-complete [32].

We were able to extend BV with the exponential modalities of linear logic, and we called NEL the system so produced. Most NEL rules are *local*, meaning that their application requires a constant effort, independent of the size of the structures involved in any particular instance. In [19, 20] we employed decomposition plus the new splitting technique introduced in [17] to prove cut elimination for NEL. The new technique appears sufficiently general to be extended to the multiplicative-additive fragment of linear logic and to classical logic (these results are currently being verified and written down). Papers [48, 49, 13] show the Turing completeness of NEL.

As a consequence, we can then get a *logical system* for distributed computation. These results, together with the results obtained by Paola Bruscoli in [11], open up the way for the first proof-theoretically induced process algebra with a pure proof-construction-computational nature. This is a subject of active research in our group (see later). In [31] Ozan Kahramanoğulları shows how the nondeterminism in system BV (and in large part also in classical logic) can be put under control, with techniques that should be useful for all the systems in the calculus of structures.

In [56, 57], we studied the unification problem for structures in BV. This opened up possibilities for implementing theorem provers based on the deep inference mechanism, especially in the non-trivial case of BV, where several different logical operators share units.

We conjecture that BV is equivalent to Retoré's pomset logic[4]. Actually, pomset logic has been the source of inspiration for my whole project on deep inference and the calculus of structures, and I am deeply indebted to Christian Retoré's intuitions.

One distinctive feature of BV is that it has been obtained by a semantic analysis on a certain kind of graphs that we call *relation webs* (they were formerly called *traces* and then *relational fields*). Relation webs allow us on one side to define the building blocks of our derivations in a

[4] Christian Retoré. Pomset logic: A non-commutative extension of classical linear logic. In Ph. de Groote and J. R. Hindley, editors, *TLCA '97*, volume 1210 of *Lecture Notes in Computer Science*, pages 300–318, 1997.

relational, non-operational style (amenable to a denotational understanding); on the other side they induce an extremely low-level, distributed computational model similar to *cellular automata*.

The abstraction level of derivations stays in the middle between those two extremes: it constitutes a skeleton for the underlying distributed computation, which ensures all the good properties entailed by cut elimination. The higher, more abstract semantic level, on the other hand, provides a non-trivial, unexpected understanding of the syntactical phenomena; this has been very useful in making and testing conjectures in this system and in all the other systems studied later.

Relation web semantics is essentially based on certain *conservation properties*, or *invariants* (for example, types are conserved in proof normalisation, i.e., they are an invariant during the execution of a functional program). We believe that conservation laws should play a much more prominent role in computation.

This research thread has two objectives for the future:

1. Studying web semantics and extending it to exponentials, in order to deal with more expressive logical systems.
2. More conceptually, attempting the introduction of conservation-based computational principles in proof construction.

Necessity of Deep Inference

The following results are contained in Alwen Tiu's master thesis (supervised by myself), successfully defended in August 2001 [58] and published in [55].

Alwen Tiu made a fundamental contribution to our research, and a remarkable invention. He found a counterexample that shows that our commutative/non-commutative systems, like BV, cannot be captured by the sequent calculus. To my knowledge, this is the only result of this kind. Since there is no definition of sequent calculus, we have to make some hypotheses, of course. Tiu makes a very mild assumption, already present in Gentzen and Prawitz: he assumes that a sequent system is *shallow*, meaning that its rules cannot be applied anywhere deep into formulae.

Alwen Tiu studied extensively what can be done by shallow and by deep systems (in the calculus of structures we can represent both kinds), and his results, aside from the one just mentioned, found applications in Paola Bruscoli's research. More in general, these results are important because they clearly show the limitations of shallow inference, and so, of the sequent calculus, especially in logics directly relevant to computer science.

2.5 Language Design (Paola Bruscoli and Ozan Kahramanoğulları)

Process Algebras as Logical Systems

The starting point of our entire research was the quest for a logical system able to match some process algebras, specifically CCS and its sequential operator. Our original system BV was in fact inspired by CCS, but what is exactly the relation?

This study is necessary for the applications, and it is by no means trivial due to the algebraic peculiarities of CCS. A certain degree of mismatch should be accepted in the end, because we cannot expect that properties meant to model actual concurrent computations (albeit abstract ones) are innocuous to cut admissibility, which is known to be a touchy subject.

What is really important is to be able to establish a bridge robust enough to carry through, both ways, the properties entailed by cut elimination and the concepts related to process algebras (bisimulation, for example).

In [11] Paola Bruscoli showed that sequentiality in CCS can be captured faithfully by sequentiality in BV. This is the only known case where a purely logical system (i.e., a system that does not use any kind of axiom, which is something that usually disrupts cut elimination) can accomplish this task.

This core system will be enriched in the future in such a way that: 1) we reach Turing completeness, and for this purpose NEL is the natural choice; 2) the correspondence to CCS is broadened to larger and larger fragments.

Planning Formalisms as Logical Systems

We are currently investigating languages of partially-ordered plans for reasoning about actions [30, 27], making use of the noncommutative features of system BV and NEL, mentioned above.

The interest of this research lies in the connection to the aforementioned research on logical languages for process algebras. We hope to be able to establish a very tight correspondence between the two fields, belonging to such disparate areas as artificial intelligence and concurrency. An expected outcome of this research will be the ability to use the very rich tool set of concurrency in the relatively poorer field of (partial order) planning.

2.6 Implementations (Steffen Hölldobler and Ozan Kahramanoğulları)

Most of the systems mentioned above are currently being implemented. There exists a web page dedicated to this at http://www.informatik.uni-leipzig.de/~ozan/maude_cos.html.

One of the most delicate problem implementations have to deal with is the abundant use of equational laws made in the theoretical investigations mentioned above. Several papers address the issues which arise, see [28, 29, 33, 40].

3 External Developments

Deep inference is increasingly adopted and studied by the community:

- Since 2002 we established a permanent collaboration with INRIA's group Calligramme in Nancy, funded by a bilateral agreement in 2003/4.
- Since 2002, our annual summer school graduate courses about deep inference draw tens of students from all over the world.
- I have been invited to teach a course on the calculus of structures at the ESSLLI 2004 summer school (probably the largest school of computer science and logic).
- The first peer-reviewed international workshop on our approach to proof theory, called 'Structures and Deduction', has been held as a satellite event of the ICALP '05 conference in Lisbon.
- The second such workshop will be held in Vienna at the margin of Gödel Centenary 2006 celebrations and conference in April 2006.
- Since 2006, we have funding for an exchange program with the PPS group in Paris about deep inference.
- Kai Brünnler has been invited to talk about deep inference at the Proofs and Computation special session of Computability in Europe in Swansea (30 June-5 July, 2006).
- I have been invited to teach a foundational course on proof theory and deep inference at the ESSLLI 2006 summer school; Lutz Straßburger has been invited at the same school with a course on proof nets and the identity of proofs.

- I have been invited as plenary speaker at the XIII Latin American Symposium on Mathematical Logic in Oaxaca (7-12 August, 2006).

The following topics have been or are being developed outside of Dresden:

3.1 Classical Logic

Kai Brünnler (Bern) shows in [5] a cut elimination procedure for predicate logic which is entirely designed inside the calculus of structures. Previous cut elimination proofs for predicate logic (as opposed to propositional classical logic) were relying on the sequent calculus.

3.2 Intuitionistic Logic

Alwen Tiu (LORIA & INRIA-Lorraine) wrote a paper, [54], where he shows a complete, cut-free and local system for intuitionistic logic.

3.3 Cyclic Linear Logic

Pietro Di Gianantonio (Udine) wrote a paper on cyclic linear logic [12] where he shows that the mysterious cyclic rule is simply avoided in a deep inference approach.

3.4 Proof Nets

Jean-Baptiste Joinet (Paris 7) presented an invited talk at WoLLIC '03 about the use of the calculus of structures with proof nets [26]. Several works on proof nets inspired by the calculus of structures [37, 53, 36, 50] have been produced in Nancy and Saarbrücken. In particular, these works propose very elegant and simple proof nets for classical logic.

3.5 Semantics of Proofs

Semantics of proofs, especially for classical logic, is one of the most exciting areas of research in proof theory. Recent contributions inspired by deep inference are: [35] by Lamarche and Straßburger; [38, 39] by Richard M^{c}Kinley at University of Bath; [25] by Dominic Hughes at Stanford University; [21, 22] by Yves Guiraud at the University of Marseille; [52] by Straßburger; [34] by Lamarche. A good introduction to the problem is [51].

4 Subatomic Formalisms and the War to Bureaucracy

We are conducting a war to bureaucracy! Fundamental progress is in [9, 16]. The objective is to design a bureaucracy-free, deductive formalism. At present, there are many deductive formalisms (all proof systems are such) and bureaucracy-free ones, like proof nets. However, no formalism has both characteristics at the same time: proof nets are not deductive. There's a strong need for such a thing, especially in view of a true geometric understanding of proofs.

In addition, there is now strong evidence that most of proof theory can be unified in such a way that inference rules are all generated by one simple scheme. This scheme generates all inference rules for all logics, and cut elimination can be studied once and for all. I call this research *subatomic proof theory*, because the fundamental idea is considering atoms as superpositions of truth states. By doing so, atoms become binary logical relations, and this gives rise to an extremely simple, universal inference rule [14, 15].

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